

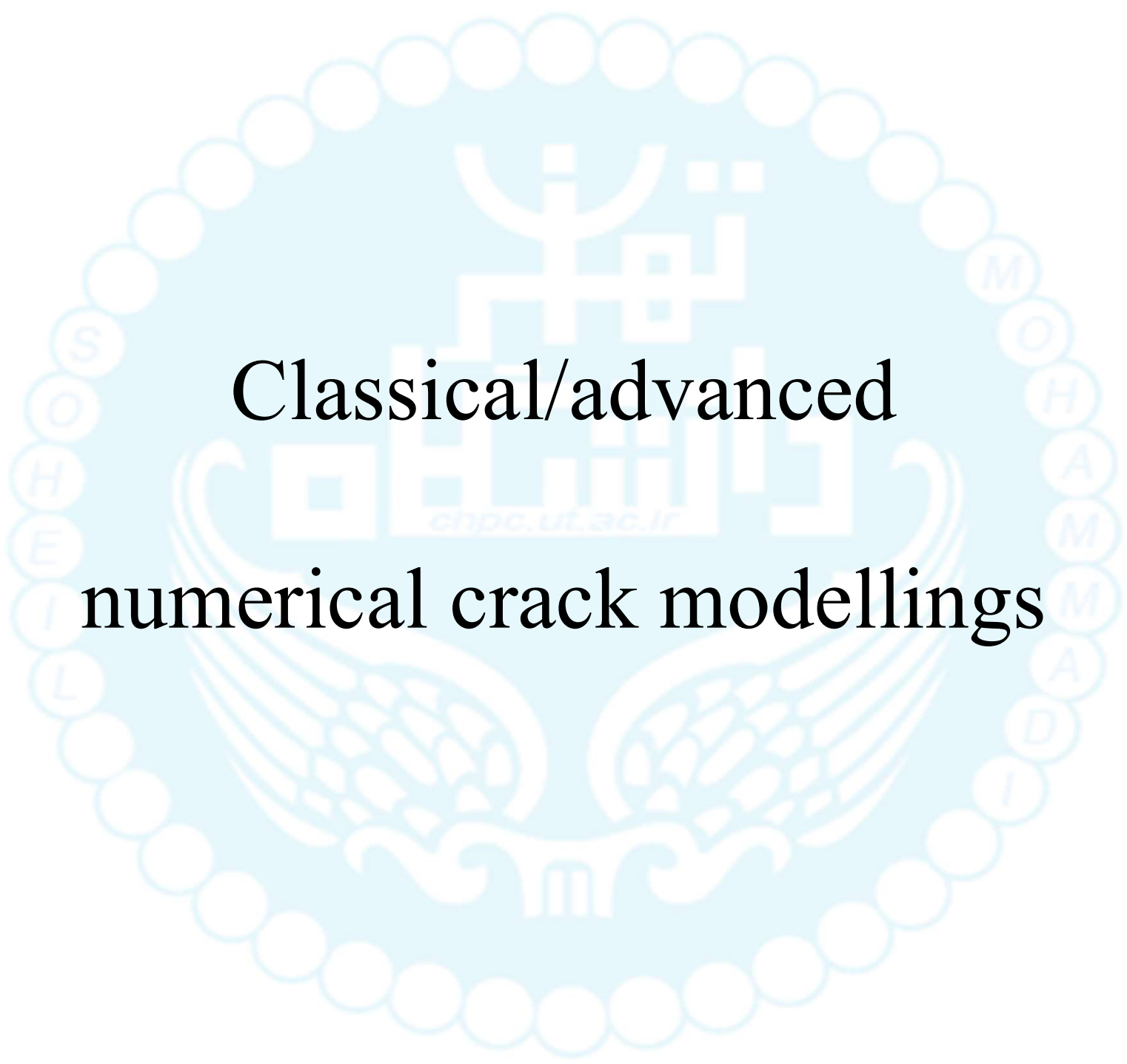


Fracture Mechanics

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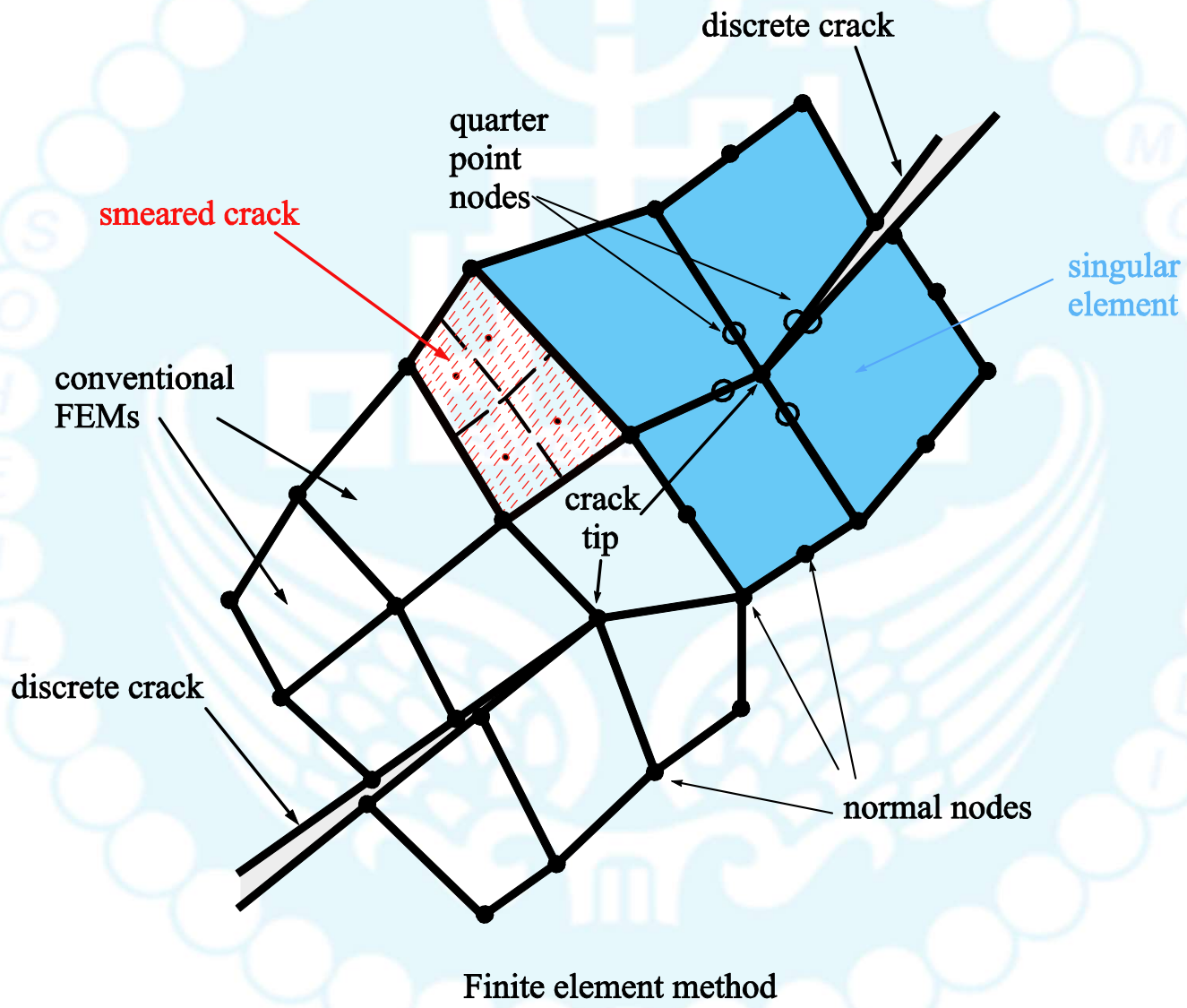
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Classical/advanced numerical crack modellings

Finite element method

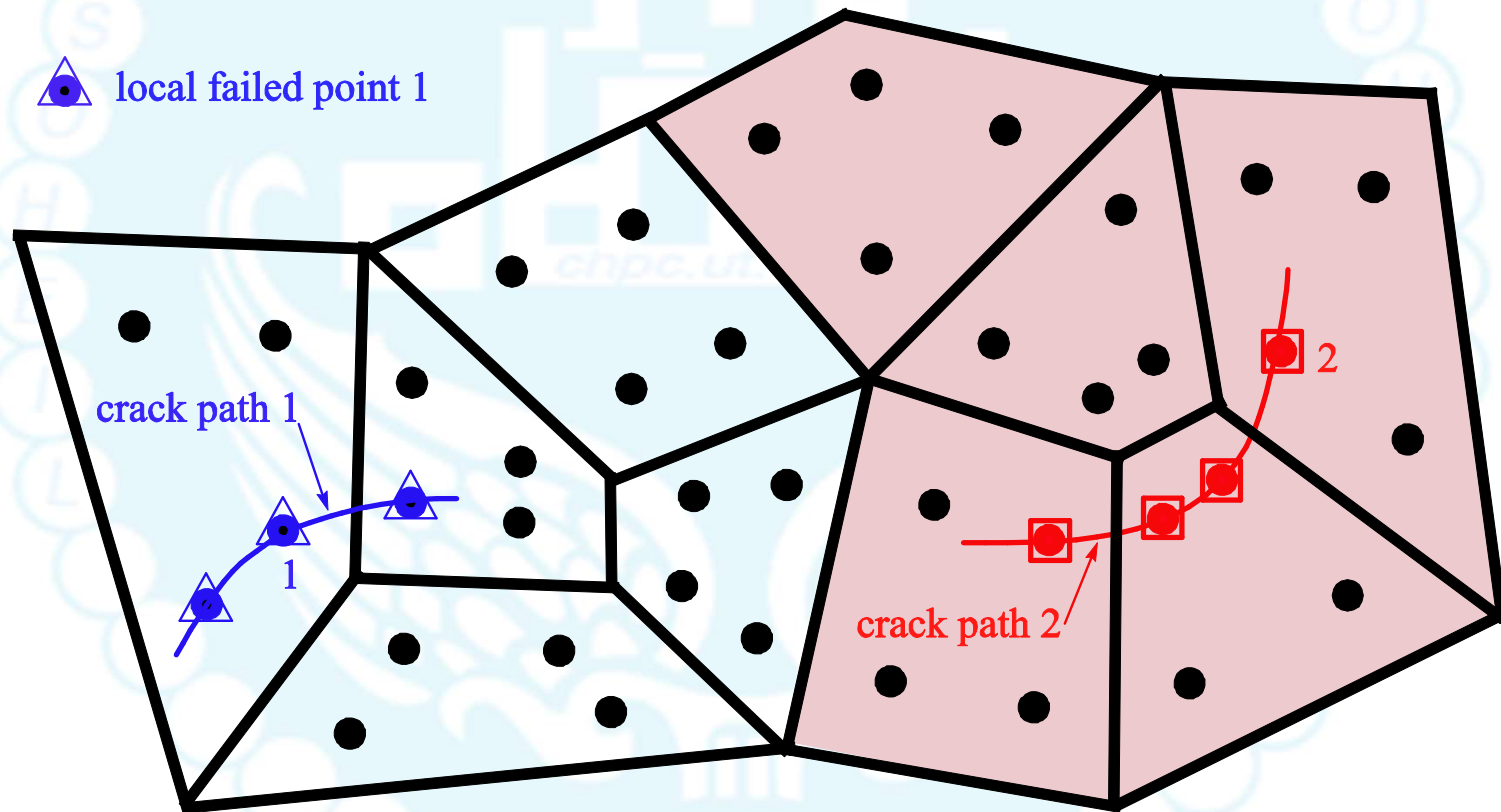


Crack modellings

● normal Gauss point

◻ non-local failed point 2

△ local failed point 1



Finite element method

Most of the performed numerical analyses of structures prior to the end of 20th century were related to the finite element method.

The finite element method can be easily adapted to complex geometries and general boundary conditions and is well developed into almost every possible engineering application, including non-linear, inhomogeneous, anisotropic, multilayer, large deformation, fracture and dynamic problems.

Several general purpose finite element softwares have been developed, verified and calibrated over the years and are now available to almost anyone who asks (and pays) for.

Furthermore, concepts of FEM are now offered by all engineering departments in the form of postgraduate and even undergraduate courses.

Finite element method

Introduction and fast development of the finite element method drastically changed the extent of application of LEFM from classical idealised models to complex practical engineering problems.

After earlier application of FEM to fracture analysis of composites (for instance by Swenson and Ingraffea, 1988) a large number of studies were adopted FEM to simply obtain the displacement, strain and stress fields required for numerical evaluation of fracture mechanics parameters such as the stress intensity factors, the energy release rate or the J integral to assess the stability of crack (Rabinovitch and Frosting, 2001, Pesic and Pilakoutas, 2003, Rabinovitch, 2004, Colombi, 2006, Lu *et al.* 2006, Yang *et al.*, 2006, Bruno *et al.*, 2007, Greco *et al.*, 2007)

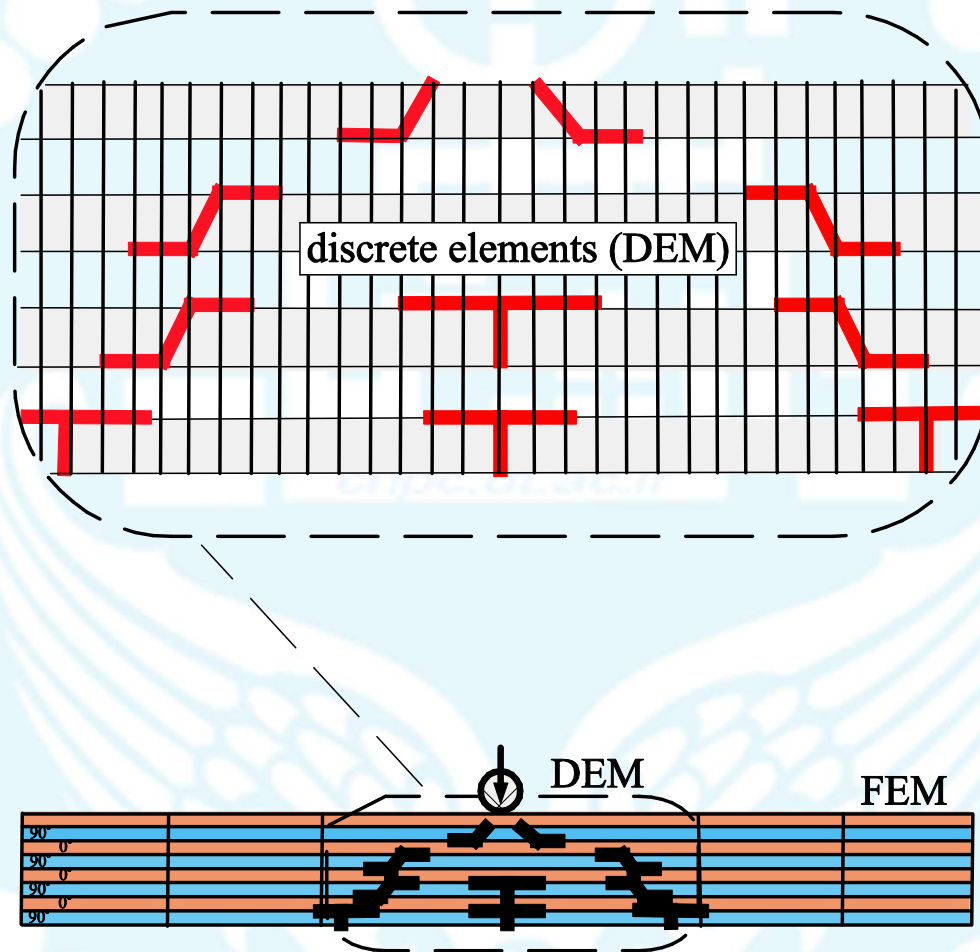
Finite element method

Later, various techniques have been developed within the FEM framework to allow for reproduction of singular stress field at a crack tip and to facilitate simulation of arbitrary crack propagations.

Singular finite elements, developed to accurately represent crack tip singular fields (Owen and Fawkes, 1983), provide the major advantage of simple construction of the model by simply moving the nearby midside nodes to the quarter points and no other changes in the finite element formulation is required (Fig. 1.4a). Prior to the development of XFEM, singular elements have been the most popular approach for fracture analysis of structures.

Singular elements, however, have to be used in a finite element mesh, where crack faces have to match element boundaries. This largely limits their application to general crack propagation problems, unless combined with at least a local adaptive finite element scheme.

Adaptive finite/discrete element method



Finite/discrete element method

Adaptive finite/discrete element method

Adaptive finite element method, combined with the concepts of contact mechanics of the discrete element method (DEM), has been adopted in several studies for simulation of progressive crack propagation in composites under quasi-static and dynamic/impact loadings. They include a variety of crack models such as smeared crack, discrete inter-element crack models, cracked interface elements and the discrete contact element, which may use general contact mechanics algorithms to simulate progressive delamination and fracture problems.

(Mohammadi *et al.*, 1997, Sprenger *et al.*, 2000, Wu *et al.*, 2002, Mohammadi and Forouzan-sepehr, 2003, Wong and Vecchio, 2003, Wu and Yin, 2003, Mohammadi, 2003, Lu *et al.*, 2005, Wang, 2006, Teng *et al.*, 2006, Mohammadi and Mousavi, 2007, Moosavi and Mohammadi, 2007, Mohammadi, 2008, Rabinovitch 2008).

Adaptive finite/discrete element method

In a composite progressive fracture analysis, part of the model, which is potentially susceptible to damage, is represented by discrete elements and the rest of the specimen is modelled with coarser finite elements to reduce the analysis time.

Each discrete element can be discretized by a finite element mesh; finer for the plies/parts closer to the damaged region and coarser elsewhere. A contact methodology controls the debonding mechanisms and all post delamination behaviours, such as sliding (Mohammadi *et al.*, 1997 and Mohammadi, 2008).

On occurrence of a crack or after a crack propagation step based on a simple comparison of the computed stress state with the adhesive strength, adaptive schemes are used to locally remesh the finite element model to ensure matching of element edges and crack faces.

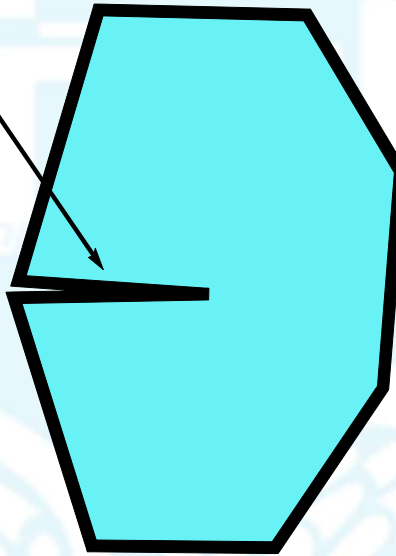
Adaptive finite/discrete element method

Non-linear material properties and geometric nonlinearities can be considered in the basic FE formulation. The method, however, is numerically expensive and the nodal alignments may cause numerical difficulties and mesh dependency to some extent in propagation problems.

The method is accurate, but numerically complex and expensive.

Boundary element method

discrete crack
boundary



Boundary element method

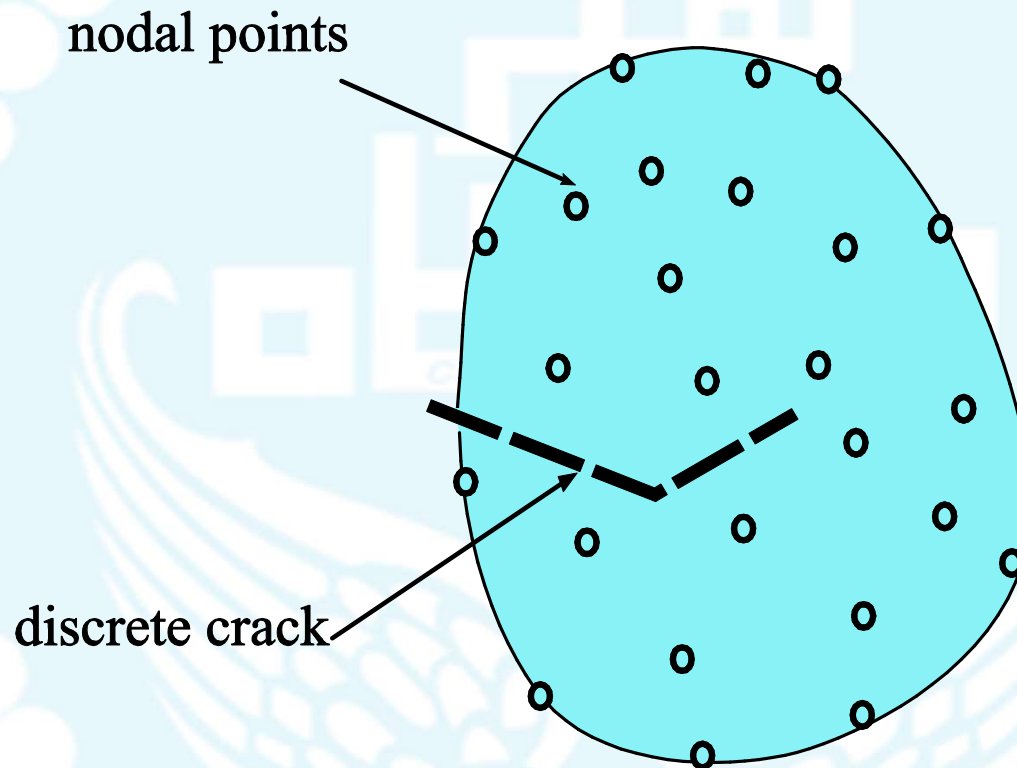
Boundary element method

Cruse (1988), Aliabadi and Sollero (1998) and García-Sánchez *et al.* (2008) developed boundary element solutions for quasi-static crack propagation and dynamic analysis of cracks in orthotropic media.

In the boundary element method, a number of elements are used to discretise the boundary of the problem domain, and the domain itself is analytical represented in the governing equations.

The boundary element method, regardless of all the benefits, cannot be readily extended to nonlinear systems and are not suited to general crack propagation problems.

Meshless methods



Meshless methods

Meshless methods

Meshless methods include a wide variety of numerical methods with different approximation techniques, diverse solution schemes, variable levels of accuracy and dissimilar applications.

Ironically, in a general view, they have nothing in common but being different from the finite element method.

The main idea of meshless methods is to avoid a predefined fixed connectivity between the nodal points which are used to define the geometry and to set necessary degrees of freedom to discretise the governing equation.

As a result of such a connection-free style of nodal discretisation, any existing cracks or crack propagation paths can be efficiently embedded geometrically within the numerical model.

Meshless methods

Meshless methods have been extensively used for analysis of various engineering problems (Belytschko *et al.*, 2002b).

They include the frequently used classes of element-free Galerkin method (EFG) (Belytschko *et al.*, 1994, Belytschko *et al.*, 1995 and Ghorashi *et al.*, 2011a), meshless local Petrov–Galerkin (MLPG) (Atluri and Shen, 2002), smoothed particle hydrodynamics (SPH) (Belytschko *et al.*, 2000, Madani and Mohammadi (2011) and Ostad and Mohammadi, 2012), finite point method (FPM) (Onate *et al.*, 1995 and Bitaraf and Mohammadi, 2010), isogeometric analysis (IGA) (Hughes *et al.*, 2005) and its extended version (XIGA) (Ghorashi *et al.*, 2012), and other approaches including the reproducing kernel particle method (RKPM) (Liu *et al.*, 1996), HP-clouds (Duarte and Oden, 1995), equilibrium on line method (ELM) (Sadeghirad and Mohammadi, 2007) and smoothed finite element method (Liu and Trung, 2010), among the others.

Meshless methods

It is noted that some of the mentioned methods are in fact a combination of finite element and meshless concepts. By this definition, the extended finite element method (XFEM) may somehow be included.

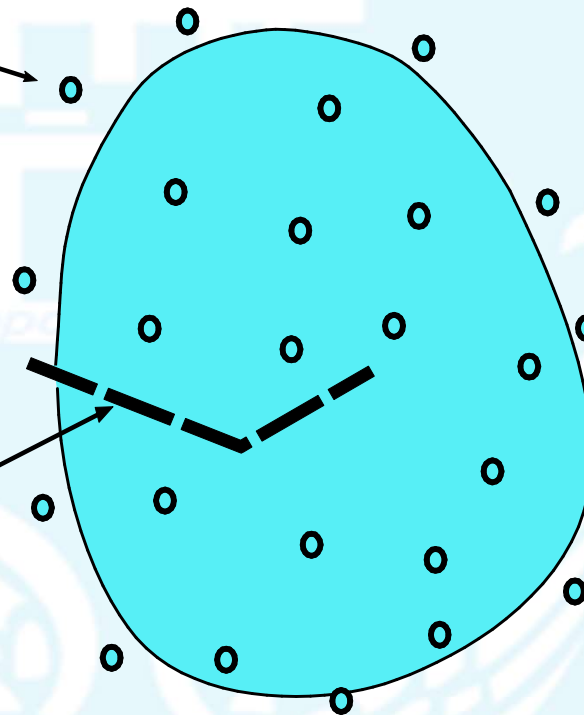
Despite higher accuracy and flexible adaptive schemes, majority of meshless methods are yet to be user friendly because of weak versatility to deal with arbitrary boundary conditions and geometries, complicated theoretical bases, high numerical expenses and the need for sensitivity analysis, calibration and difficult stabilization schemes in many of them.

Extended isogeometric analysis

control points

discrete crack

Extended IGA (XIGA)



Extended isogeometric analysis

Since the introduction of the IsoGeometric Analysis (IGA) by Hughes *et al.* (2005) a new and fast growing chapter has been opened in unifying the computer aided design (CAD) and the numerical solutions by using the non-uniform rational B-splines (NURBS) functions. The method, in fact, can be categorized with other meshless methods, but it is briefly introduced due to its fast growing state and excellent potentials.

Extended isogeometric analysis

IGA has been successfully adopted in several engineering problems, including structural dynamics (Cottrell *et al.*, 2009 and Hassani *et al.*, 2009), Navier–Stokes flow (Nielsen *et al.*, 2011), fluid-solid interaction (Bazilevs *et al.*, 2009), shells (Benson *et al.*, 2010a, Benson *et al.*, 2011, Kiendl *et al.*, 2010, Kiendl *et al.*, 2009), damage mechanics (Verhoosel *et al.*, 2010a), cohesive zone simulations (Verhoosel *et al.*, 2010b), heat transfer (Anders *et al.*, 2012), large deformation (Benson *et al.*, 2011), electromagnetic (Buffa *et al.*, 2010), strain localization (Elguedj *et al.*, 2011), contact mechanics (Lu, 2011 and Temizer *et al.*, 2011), Topology optimization (Hassani *et al.*, 2012) and crack propagation (Verhoosel *et al.*, 2010b, Benson *et al.*, 2010b, De Luycker *et al.*, 2011, Haasemann *et al.*, 2011 and Ghorashi *et al.*, 2012).

Extended isogeometric analysis

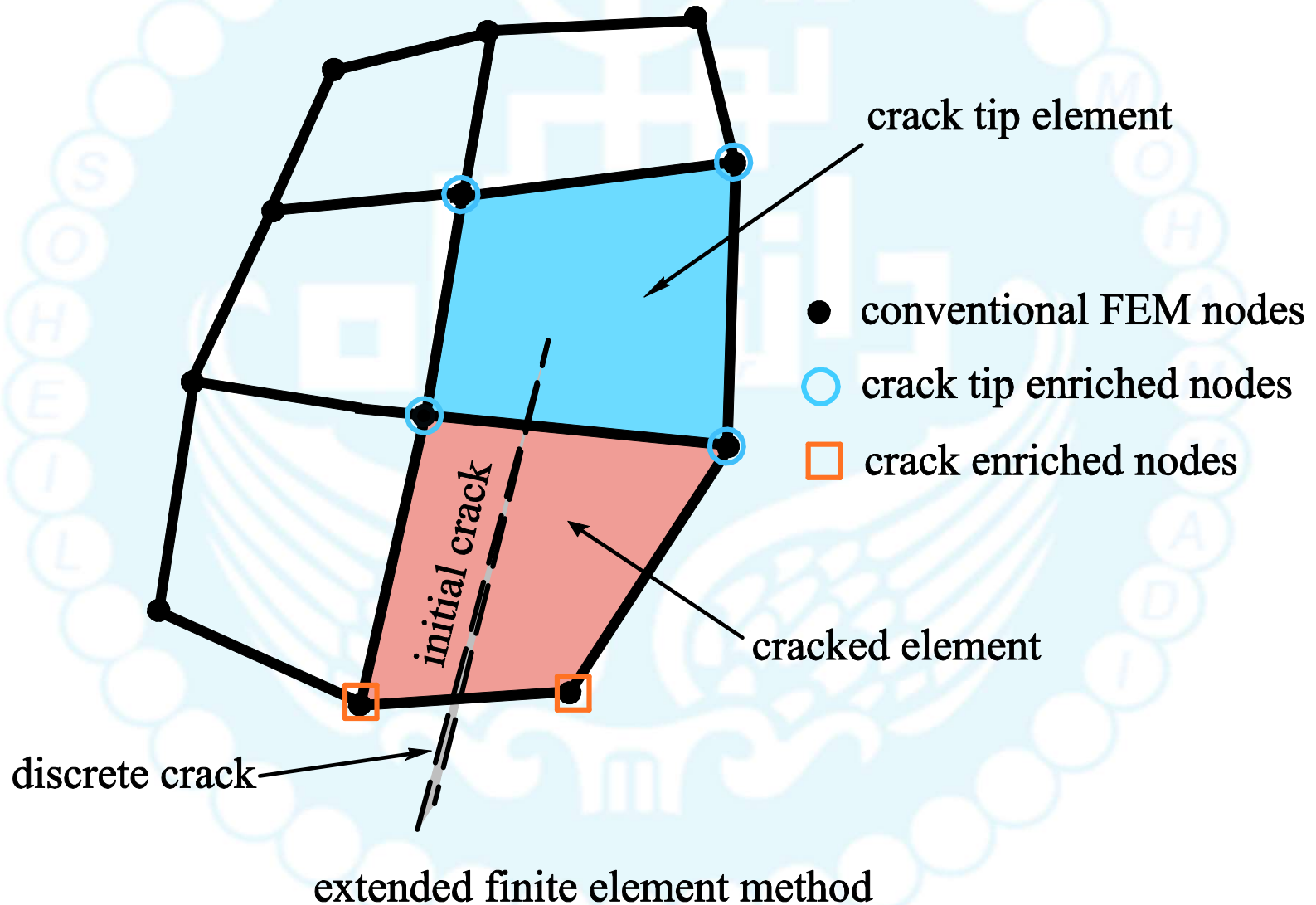
The first attempt for enhancing IGA for crack problems was reported by Verhoosel *et al.* (2010b) and followed by Benson *et al.* (2010b), Luycker *et al.* (2011) and Haasemann *et al.* (2011).

More recently, a full combination of XFEM and IGA methodologies has been developed for general mixed mode crack propagation problems by the introduction of eXtended IsoGeometric Analysis (XIGA) by Ghorashi *et al.* (2012).

XIGA uses the superior concepts of XFEM to extrinsically enrich the versatile IGA control points with Heaviside and crack tip enrichment functions.

The method is numerically complex.

Extended finite element method



Extended finite element method

Since the introduction of the extended finite element method (XFEM) for fracture analysis by Belytschko and Black (1999), Moës *et al.* (1999) and Dolbow (1999) based on the mathematical foundation of the partition of unity finite element method (PUFEM), proposed earlier by Melenk and Babuska (1996) and Duarte and Oden (1996), XFEM methodology has been rapidly extended to a vastly wide range of applications that somehow include a local discontinuity or singularity within the solution.

Extended finite element method

The natural extension of FEM into XFEM allows for new capabilities while preserving the finite element original advantages. The two main superiorities of XFEM are its capability in reproducing the singular stress state at a crack tip, and allowing for several cracks or arbitrary crack propagation paths to be simulated on an independent unaltered mesh.

In fact, while the presence of the crack is not geometrically modelled and the mesh does not need to conform to the virtual crack path, the exact analytical solutions for singular stress and discontinuous displacement fields around the crack (tip) are reproduced by inclusion of a special set of enriched shape functions that are extracted from the asymptotic analytical solutions.

Extended finite element method

Apart from earlier works that were directed towards the development of the extended finite element method for linear elastic fracture mechanics (LEFM), simulation of failure and localisation has been the target of several studies including, including Jirásek and Zimmermann (2001a, 2001b), Sukumar *et al.*, 2003b, Dumstorff and Meschke, 2003 Patzak and Jirásek, 2003, Ventura *et al.*, 2005, Samaniego and Belytschko (2005), Areias and Belytschko (2005a,b and 2006) and Song *et al.* (2006). In addition, various contact problems have been simulated by XFEM. For example, Dolbow *et al.* (2000c, 2001), Shamloo *et al.* (2005), Belytschko *et al.* (2002a), Khoei and Nikbakht (2006), Khoei *et al.* (2006a, 2006b) have used simple Heaviside enrichments to deal with contact discontinuities in different problems, while Ebrahimi *et al.* (2012) have recently proposed a numerical approach within the partition of unity finite element method to determine the order of singularity at a sliding contact corner.

Extended finite element method

Moreover, several dynamic fracture problems have been studied by XFEM. Among them, Peerlings *et al.* (2002), Belytschko *et al.* (2003), Oliver *et al.* (2003), Ventura *et al.* (2003), Chessa and Belytschko (2004, 2006), Belytschko and Chen (2004), Zi *et al.* (2005), Rethore *et al.* (2005a,c), Menouillard *et al.* (2006), Gregoire *et al.* (2007), Nistor *et al.* (2008), Prabel *et al.* (2008), Combescure *et al.* (2008), Gregoire *et al.* (2008), Motamedi (2008), Kabiri (2009), Gravouil *et al.* (2009a, 2009b) and Rezaei (2010) have discussed various aspects of general dynamic fracture problems.

Extended finite element method

In the past decade, development of XFEM has substantially contributed to new studies of fracture analysis of various types of composite materials. Dolbow and Nadeau (2002) employed XFEM to simulate fracture behaviour of micro-structured materials with a focus on functionally graded materials. Then, Dolbow and Gosz (2002) described a new interaction energy integral method for the computation of mixed mode stress intensity factors in functionally graded materials.

Extended finite element method

Remmers *et al.* (2003) presented a new formulation for delamination in thin-layered composite structures. Study of bimaterial interface cracks was performed by Sukumar *et al.* (2004) by developing new bimaterial enrichment functions. Nagashima *et al.* (2003) and Nagashima and Suemasu (2004, 2006) described the application of XFEM to stress analyses of structures containing interface cracks between dissimilar materials, and concluded the need for orthotropic enrichment functions to represent the asymptotic solution for a crack in an orthotropic material. Hettich and Ramm (2006) simulated the delamination crack as a jump in the displacement field without using any crack tip enrichment.

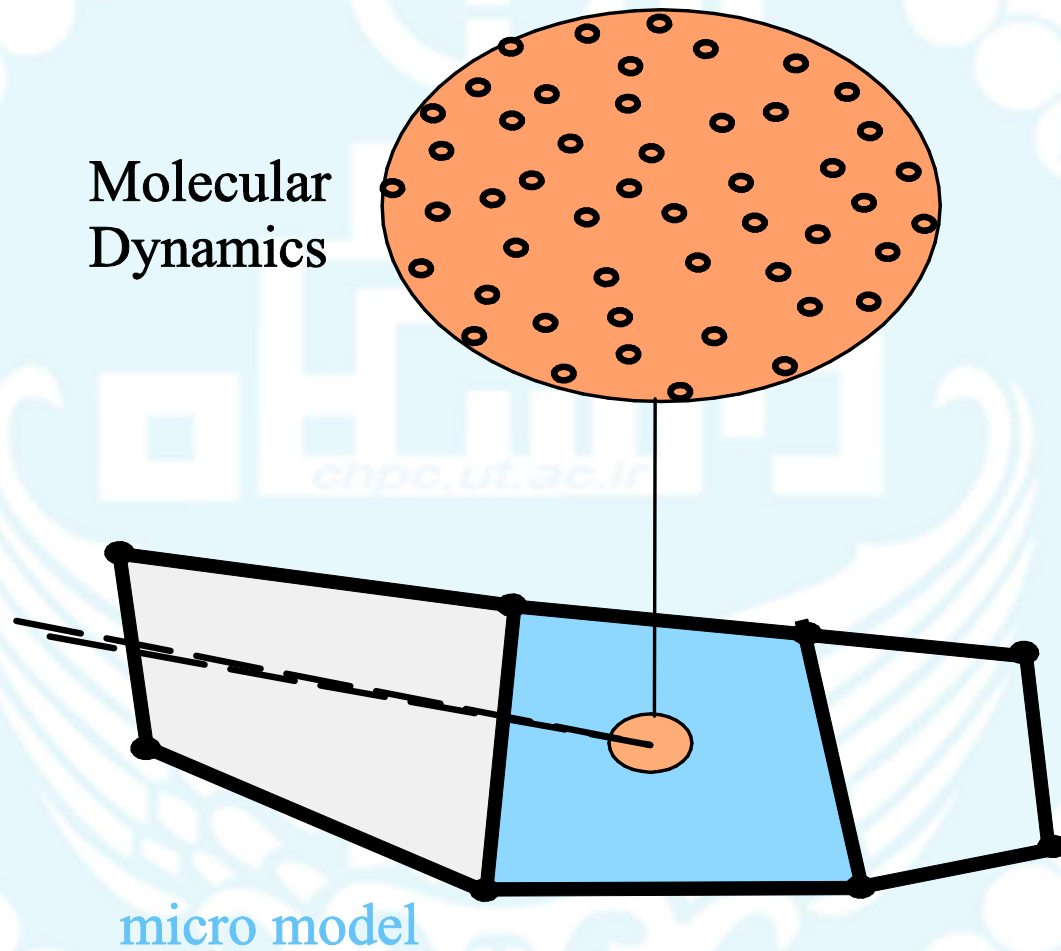
Extended finite element method

Also, a number of XFEM simulations have focused on thermo-mechanical analysis of orthotropic FGMs (Dag *et al.*, 2007), 3D isotropic FGMs (Ayhan, 2009, Zhang *et al.*, 2011 and Moghaddam *et al.*, 2011) and frequency analysis of cracked isotropic FGMs (Natarajan *et al.*, 2011). Recently, Bayesteh and Mohammadi (2012) have used XFEM with orthotropic crack tip enrichment functions to analyse several FGM crack stability and propagation problems.

Extended finite element method

Development of independent orthotropic crack tip enrichment functions was reported in a series of papers by Asadpoure *et al.* (2006, 2007), Asadpoure and Mohammadi (2007) and Mohammadi and Asadpoure (2006). Later, Motamedi (2008) and Motamedi and Mohammadi (2010a, 2010b, 2012) studied the dynamic crack stability and propagation in composites based on static and dynamic orthotropic enrichment functions and Esna Ashari (2009) and Esna Ashari and Mohammadi (2009, 2010, 2011a and 2012) have further extended the method for orthotropic bimaterial interfaces.

Multiscale analysis



Multiscale simulation

Multiscale analysis

The traditional borders between simulation of mechanics, physics and even biology problems have been removed by recent computational advances in the form of multiscale simulations. Most problems in science involve many scales in time and space. An example is the overall stress state in a cracked solid which can well be described by macroscopic continuum equations, but requires details on a microscale at the tip of the crack.

In a multiscale method, part of the model which requires a more accurate theoretical basis or numerical approximation, due to lack of theoretical bases or existing inconsistencies of many conventional models, is simulated by a finer modelling scale, which can better represent details of the material behaviour and the interacting effects of material constituents.

Multiscale analysis

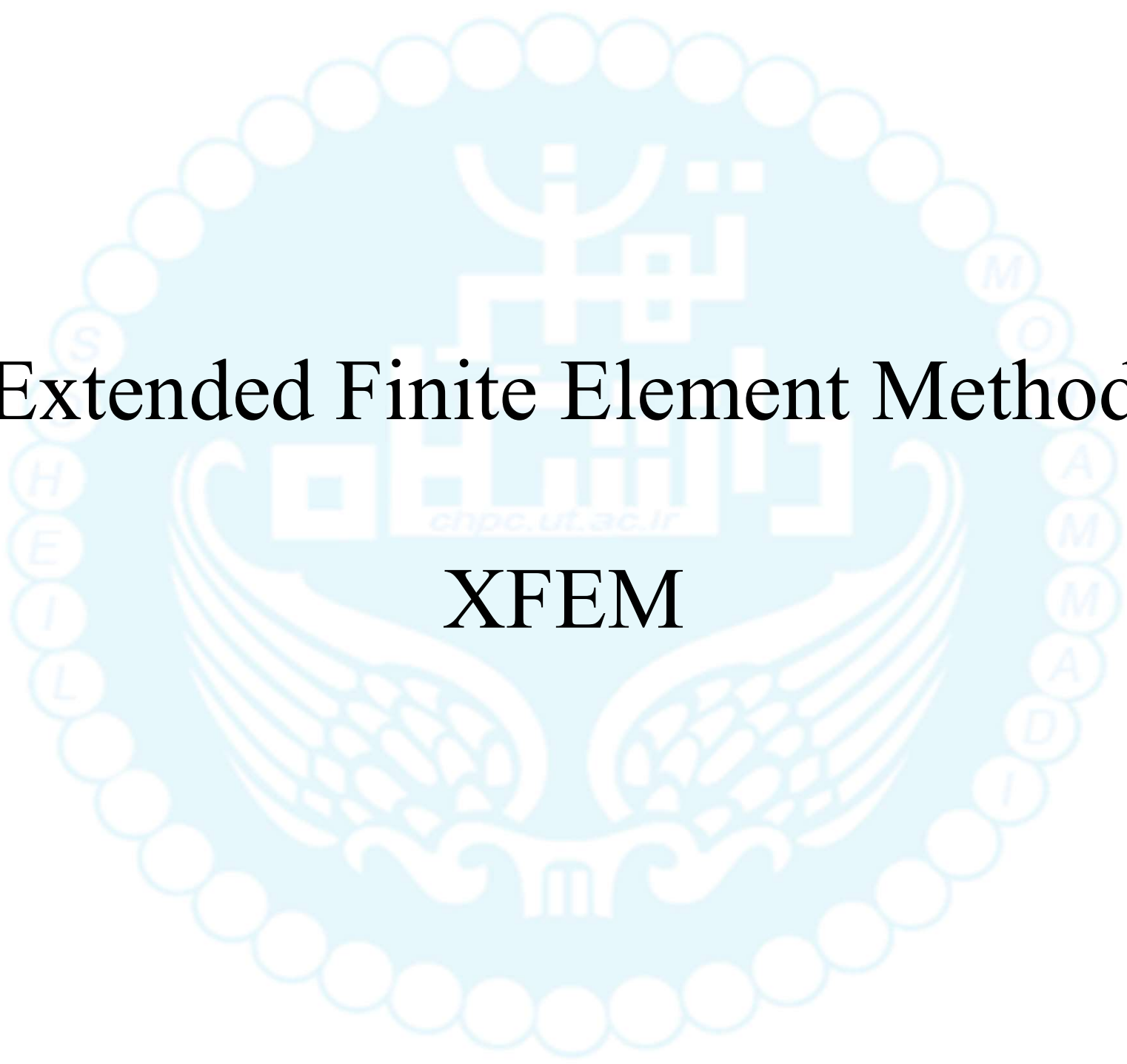
A common difficulty with the simulation of these problems and many others in physics, chemistry, engineering and biology is that an attempt to represent all fine scales will lead to an enormous computational model with unacceptably long computation times and huge memory requirements, even for state of the art modern supercomputers.

On the other hand, if the discretization at a coarse level ignores the fine scale information then the solution may not be physically meaningful. Therefore, the challenging task of incorporating the influence of the fine scales into the coarse model must logically be determined.

Multiscale analysis

Several multiscale techniques have been developed for different applications, including the Bridging Scale Method, the Bridging Domain Method, Homogenization Techniques, Quasi Continuum Method, Heterogeneous Multiscale Method, etc.

These methods can be incorporated into multiscale FEM, meshless or XFEM methodologies to analyse various multiscale problems at macro/micro, micro/nano, macro/nano, macro/micro/nano and bio/nano interfaces levels (Mohammadi, 2012).



Extended Finite Element Method

XFEM



XFEM

Introduction,
literature review

The first enriched finite element model was proposed by Benzley (1974) who introduced the idea of enriching the near tip field in the finite element method using the asymptotic solution for static fracture problems.

This work was continued by Atluri *et al.* (1975a,b) and Gifford and Hilton (1978) for stationary cracks. It took, however, until 1999 for a first practical enriched finite element method to be developed for general crack propagation problems in the form of the extended finite element method.

The goal has been set to describe in detail all theoretical and computational bases of the approach, including advantages and potential drawbacks.

The discussion is limited to two-dimensional XFEM, although many of the basic formulations can be similarly extended to three-dimensional problems.

The introduction section begins with a broad review of general developments of the XFEM methodology, and will then be followed by specific examination of various composite applications, whose details will be comprehensively discussed in the next five chapters.

The main part of the chapter starts with the theoretical discussion on fundamental bases of the partition of unity and the existing extensions to more advanced frameworks, primarily based on fracture mechanics applications.

The concept of enrichment will then be explained in detail based on the concept of partition of unity and the extended finite element method. It covers the basic techniques and definitions of enrichment functions, comprising crack tip enrichments to reproduce asymptotic crack tip fields, jump functions to approximate field discontinuity across a crack, and weak discontinuity functions to simulate a material discontinuity interface within a finite element.

The XFEM approximation is then implemented to discretise the weak formulation of the boundary value problem and various standard and enriched components of the discretised stiffness matrix and the force vector are discussed.

The section continues with addressing numerical implications regarding the Gauss quadrature rules, partitioning methods, etc.

Three comprehensive sections are devoted to discussing various enrichment functions. First, different options of the strong discontinuity enrichments are described, which include conventional definitions, tangential discontinuity, intersecting cracks, etc. In the next section, the problem of weak discontinuity is analysed and appropriate enrichments are defined.

Then, a very comprehensive list of crack tip enrichments in a wide-range of applications, from isotropic functions to orthotropic and inhomogeneous media, contact, viscoelasticity, piezoelectric applications, and dislocations within the static, dynamic and large deformation regimes, are presented and explained.

A brief section on tracking evolving boundaries, such as crack propagation paths, is necessary for most XFEM simulations. Among the existing algorithms, the Level set method, as a powerful alternative to classical approaches for representing crack paths and crack intersections, is examined.

The chapter concludes with simulation and discussion of several engineering examples to illustrate the validity, robustness and efficiency of XFEM in dealing with various discontinuity and singularity problems. Specific problems related to cracking in composites will be dealt with in subsequent chapters.

HISTORIC DEVELOPMENT OF XFEM

Despite the innovative idea of enriching the near tip field in the finite element method by Benzley (1974), Atluri *et al.* (1975) and Gifford *et al.* (1978), fundamentals of partition of unity finite element method and the extended finite element method were developed twenty years later by Melenk and Babuska (1996) and Belytschko and Black (1999). In less than a decade, XFEM has now become the main numerical approach for solution of all kind of discontinuity problems.

Melenk and Babuska (1996) discussed the mathematical bases of the partition of unity finite element method (PUFEM) and illustrated that PUFEM can be used to employ the structure of the differential equation under consideration to construct effective and robust methods. The same idea of PUFEM, but in a local nature, has become the theoretical basis of the local partition of unity finite element method, to be called later the extended finite element method.

The first effort for developing the extended finite element methodology was reported by Belytschko and Black (1999), presenting a minimal remeshing finite element method for crack growth. They added discontinuous enrichment functions to the finite element approximation to account for the presence of the crack. A local remeshing scheme was only required for severely curved arbitrary cracks to improve the solution.

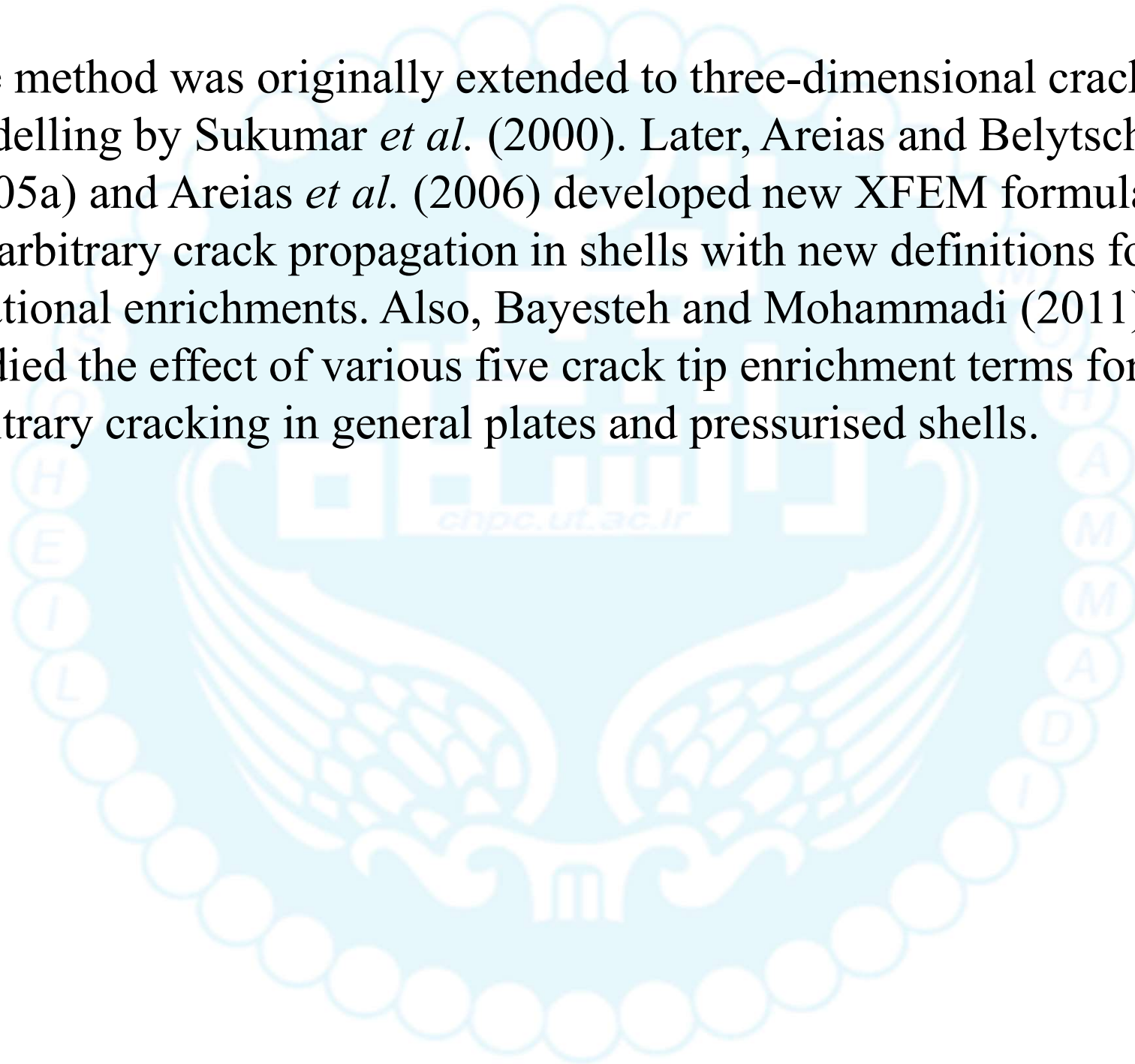
The terminology of the extended finite element method (XFEM) was proposed by Moës *et al.* (1999), as they improved the methodology by allowing for mesh independent representation of the entire crack based on the construction of the enriched approximation.

Another major step forward was achieved by Dolbow (1999) and (Dolbow *et al.* 2000a, 2000b, 2000c) for XFEM simulation of two-dimensional elasticity and Mindlin–Reissner plates using both a jump function and the asymptotic near tip fields. They also presented a technique to model arbitrary discontinuities in the finite element framework by locally enriching a displacement based approximation through a partition of unity method.

Further development was performed by Daux *et al.* (2000) by simulation of arbitrary branched and intersecting cracks with multiple branches, multiple holes and cracks emanating from holes, Sukumar and Prevost (2003), Huang *et al.* (2003) and Legay *et al.* (2005) who extended XFEM to simulation of strong and weak discontinuities, Chessa *et al.* (2003) by discussing the construction of blending elements, Lee *et al.* (2004) for modelling of stationary and growing cracks by superposition of s-version FEM with XFEM, and Stazi *et al.* (2003) formulating higher order elements for curved cracks.

Also, Liu *et al.* (2004) proposed an improved XFEM by including higher order terms of the crack tip asymptotic field in the enrichment approximation and applying a penalty function method to ensure the reproduction of the actual asymptotic fields. In addition, they adopted a direct approach for evaluation of mixed mode stress intensity factors (SIFs) without extra post processing.

The method was originally extended to three-dimensional crack modelling by Sukumar *et al.* (2000). Later, Areias and Belytschko (2005a) and Areias *et al.* (2006) developed new XFEM formulations for arbitrary crack propagation in shells with new definitions for rotational enrichments. Also, Bayesteh and Mohammadi (2011) studied the effect of various five crack tip enrichment terms for arbitrary cracking in general plates and pressurised shells.



Generalisation of XFEM for arbitrary crack propagation problems required implementation of tracking techniques for moving boundaries.

The level set method (LSM) has since become the primary approach for determining the crack path and the location of crack tips. Stolarska *et al.* (2001) introduced coupling the level set method with XFEM to model crack growth and Belytschko *et al.* (2001) proposed a technique for approximating arbitrary discontinuities in the main field and its derivatives in terms of a signed distance function, so level sets could be used to update the position of the discontinuities.

A similar approach was adopted by Sukumar *et al.* (2001) to model holes and inclusions by level sets within an XFEM framework. An extension to non-planar three dimensional crack growth was developed by Moës *et al.* (2002) and Gravouil *et al.* (2002) based on a Hamilton–Jacobi equation to update the level sets with a velocity extension approach to preserve the old crack surface.

Budyn *et al.* (2004) presented a combined XFEM/level set method for modelling homogeneous and inhomogeneous linear elastic media and Zi *et al.* (2004) discussed the junction of two cracks and analysed the growth and mixture of cracks in a quasi-brittle cell containing multiple cracks.

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The level set method has not been the sole tracking technique within XFEM. The fast marching method (FMM) was coupled by the extended finite element method to develop a numerical technique for planar three-dimensional fatigue crack growth simulations (Sukumar *et al.*, 2003a).

Chopp and Sukumar (2003) extended the method to represent the entire multiple coplanar crack geometry by a single signed distance (level set) function. The method allowed for merging of distinct cracks by FMM with no necessity of collision detection or mesh reconstruction procedures.

A different approach for tackling the same set of problems was proposed by Ventura *et al.* (2003).

Following the initial success of the method, XFEM has been extensively applied in various engineering applications. The problem of cohesive cracks was studied by Moës and Belytschko (2002a), Zi and Belytschko (2003), Mergheim *et al.* (2005), Forghani (2005), de Borst *et al.* (2004a, 2004b, 2004c), and Mahdavi and Mohammadi (2012), among the others. Also, Bellec and Dolbow (2003) and Mariani and Perego (2003) adopted XFEM for simulation of cohesive crack propagation by assuming a cubic displacement discontinuity that allowed reproduction of the typical cusp-like shape of the process zone at the tip of a cohesive crack. Areias and Belytschko (2005a, 2005b) coupled XFEM with a viscosity-regularised continuum damage constitutive model to develop a regularised ‘crack-band’ version of XFEM. Recently, Mahdavi and Mohammadi (2012) have adopted a cohesive hydraulic crack model to simulate dynamic crack propagation in concrete dams.

Application of XFEM for crack analysis in composites started by Dolbow and Nadeau (2002), Dolbow and Gosz (2002), Remmers *et al.* (2003), Sukumar *et al.* (2004), Nagashima *et al.* (2003) and Nagashima and Suemasu (2004), and then followed by Asadpoure *et al.* (2006), Asadpoure *et al.* (2007) and Asadpoure and Mohammadi (2007) by introducing new orthotropic enrichment functions, a beginning for future development of orthotropic enrichment functions in various applications, as presented in Section 3.2.2.

The localisation problem has been studied by several authors including Jirásek and Zimmermann (2001a, 2001b), Jirásek (2002), Sukumar *et al.* (2003b), Patzak and Jirásek (2003), Dumstorff and Meschke (2003), Samaniego and Belytschko (2005), Areias and Belytschko (2006), Song *et al.* (2006), Ferrie *et al.* (2006), Ventura *et al.* (2005), Larsson and Fagerström (2005) and Stolarska and Chopp (2003). Recently, Daneshyar and Mohammadi (2011, 2012a,b) have developed a strong tangential discontinuity model for XFEM simulation of shear band initiation and propagation.

Modelling contact by XFEM was first introduced by Dolbow *et al.* (2000c, 2001) and revisited by Belytschko *et al.* (2002a). Khoei and Nikbakht (2006, 2007) applied the available formulation to modelling frictional contact problems. In addition, Ebrahimi *et al.* (2012) have recently proposed a numerical approach for determining the order of singularity of a sliding contact corner within the framework of partition of unity finite element method.

Introduction of plastic enrichment terms into XFEM was first reported by Elguedj *et al.* (2006) based on the Hutchinson–Rice–Rosengren (HRR) fields to represent the singularities in EPFM. Khoei *et al.* (2008a,b,c) have simulated several plasticity problems by XFEM. Further development into viscoelastic regime was reported by Zhang *et al.* (2010) and TianTang and QingWen (2011) and followed by Hajikarimi *et al.* (2012) for simulation of crack healing in pavement applications.

Many researchers have tackled the XFEM within large deformation regime, among them Dolbow and Devan (2004), Legrain *et al.* (2005), Mergheim *et al.* (2006), Fagerström and Larsson (2006), Anahid and Khoei (2008), Khoei *et al.* (2008a-c) and Rashetnia *et al.* (2012), presenting various aspects of geometrically nonlinear XFEM analyses.

Damage mechanics has recently attracted so much attention within the XFEM community (Duarte, 2011, Jin *et al.*, 2011, Moes *et al.*, 2011). Combescure *et al.*, 2011 discussed different problems of transition from brittle damage to ductile fracture with cohesive XFEM, and Moes (2011) proposed a thick level set approach for modelling damage growth and its transition to fracture. Also, Broumand and Khoei (2011) combined the damage plasticity and XFEM to simulate ductile fracture problems and Hatefi and Mohammadi (2012) adopted XFEM for damage analysis of orthotropic materials.

XFEM has been extensively adopted for dynamic fracture analysis. Dynamic XFEM was proposed by Belytschko *et al.* (2003), Belytschko and Chen (2004) and Zi *et al.* (2005) based on the singular enrichment finite element method for elastodynamic crack propagation. Also, Réthoré *et al.* (2005a) proposed a generalised XFEM to model dynamic fracture and time dependent problems. Later, Menouillard *et al.* (2006) presented an explicit XFEM by introduction of a lumped mass matrix for enriched elements. In addition, Motamedi and Mohammadi (2010a,b and 2012) developed new orthotropic enrichment functions for moving cracks, and Parchei *et al.* (2012) used the dynamic XFEM combined with a contact approach to simulate near field solution for various induced fault rupture problems.

The concept of spatial discontinuity enrichment has also been extended to the time domain by introduction of a locally enriched space–time extended finite element method by Chessa and Belytschko (2004, 2006) or by solving hyperbolic problems with discontinuities. The coupling was implemented through a weak enforcement of the continuity of the flux between the space–time and semi-discrete domains in a manner similar to discontinuous Galerkin methods. They successfully applied the TXFEM to the Rankine–Hugoniot jump conditions for linear first order wave and nonlinear Burgers' equations. Furthermore, Réthoré *et al.* (2005b) proposed a combined space–time extended finite element method, based on the idea of the time extended finite element method, allowing a suitable form of the time stepping formulae to study stability and energy conservation. The proposed TXFEM was successfully adopted by Kabiri (2009) and Rezaei (2010) for simulating various dynamic fracture problems.

A number of XFEM investigations have been directed towards the study of accuracy, stability and convergence. Laborde *et al.* (2005) and Chahine *et al.* (2006) studied the convergence of a variety of XFEM models on cracked domains. Peters and Hack (2005) discussed the ways that a singular stiffness matrix may be avoided by deleting some of the enhanced degrees of freedom, while Béchet *et al.* (2005) proposed a geometrical enrichment instead of the usual topological one in which a given domain size would be enriched even if the elements did not touch the crack front. Ventura (2006) showed how the standard Gauss quadrature rule can be accurately used in the elements containing the discontinuity without splitting the elements into subdomains or introducing any additional approximation.

Xiao and Karihaloo (2006) discussed improving the accuracy of XFEM crack tip fields using higher order quadrature and statically admissible stress recovery procedures. They proposed a statically admissible stress recovery (SAR) scheme to be constructed by basis functions and moving least squares (MLS) to fit the stresses at sampling points (e.g. quadrature points) obtained by XFEM.

In addition to the first text book on the subject (Mohammadi, 2008), there are also a number of available review papers and comprehensive references such as Moës and Belytschko (2002b), Karihaloo and Xiao (2003), Bordas and Legay (2005), Rabczuk and Wall (2006), Chahine (2009) and Pommier *et al.* (2011), which can be referred to for further details.

A review of XFEM composite analysis

The first applications of XFEM for composites was reported by Dolbow and Nadeau (2002), Dolbow and Gosz (2002) and Remmers *et al.* (2003), while Nagashima and Suemasu (2004, 2006) simulated thin layered composites and concluded that the isotropic enrichment functions cannot represent the asymptotic solution for a crack in an orthotropic material.

Then, Sukumar *et al.* (2004), developed partition of unity based enrichment functions for bimaterial interface cracks between two isotropic media, as a special type of multilayer composites. Later, Hettich and Ramm (2006) simulated the delamination crack as a jump in the displacement field without using any crack tip enrichment.

Also, a number of XFEM simulations have focused on thermo-mechanical analysis of orthotropic FGMs (Dag *et al.* 2007), 3D isotropic FGMs (Ayhan 2009, Zhang *et al.* 2011, Moghaddam *et al.* 2011) and frequency analysis of cracked isotropic FGMs (Natarajan *et al.* 2011). The most recent work is reported by Bayesteh and Mohammadi (2012) on general crack propagations in orthotropic FGM composites. They have discussed the effect of crack tip enrichments and various types of interaction integral for determining the mixed mode stress intensity factors.

Development of orthotropic crack tip enrichment functions was reported in a series of papers by Asadpoure *et al.* (2006, 2007), Asadpoure and Mohammadi (2007) and Mohammadi and Asadpoure (2006). They developed three types of enrichment functions for various classes of composites. Ebrahimi *et al.* (2008) used the same enrichments to analyse several composite crack propagation problems. Similar enrichment functions were implemented in meshless EFG method by Ghorashi *et al.* (2011a) to analyse crack propagation in composites.

Later, Motamedi (2008) and Motamedi and Mohammadi (2010a, 2010b) studied the dynamic crack stability and propagation in composites based on static orthotropic enrichment functions, while Motamedi and Mohammadi (2012) developed new orthotropic dynamic enrichment functions for moving cracks in orthotropic composites.

In addition, Esna Ashari (2009) and Esna Ashari and Mohammadi (2009, 2010, 2011a,b and 2012) further extended the method for orthotropic bimaterial interfaces. They also adopted the same functions to analyse delamination in FRP-strengthened structures and to predict the crack arrest mechanism and the load bearing capacity in terms of the debonded length.

